S.5 MATHEMATCIS PAPER ONE

CHAPTER ONE : DIFFERENTIATION

REVISION QUESTIONS 2020

Attempt all questions

1. Find the gradient function $\frac{dy}{dx}$ for each of the following functions.
   a) $y = x^2 + 7x - 4$
   b) $y = 6x^2 - 7x + 8$
   c) $y = 3x^6 - 7x^2 + 6x - 8$
   d) $y = 3x - \frac{5}{x} + \frac{6}{x^2}$
   e) $(x^7+2)(4x-1)$

2. Find the gradients of the following lines at the points indicated.
   a) $y=2x^3-x^2 + 3x - 1$ at (-1,-6)
   b) $y = x^2 + 7x - 4$ at (2,21)
   c) $y = 2x^2 - x + \frac{4}{x}$ at (2,8)
   d) $y = 3x + \frac{1}{x}$ at (1,4)
   e) $y = (x+1)(2x + 3)$ at (2,21)

3. If $f(x) = x^3 + 4x$ find
   a) $f(1)$
   b) $f'(x)$
   c) $f'(1)$
   d) $f''(x)$

e) $f''(2)$

4. If $f(x) = 3x^2 + \frac{24}{x}$ find

   a) $f'(x)$

   b) $f'(-12)$

5. Find the equation of the normal to the curve $y=x^2 + 4x - 3$ at the point where the curve cuts the y- axis. Ans : $4y + x + 12 = 0$

6. Find the equation of the tangent to the curve $y=x^2 - 3x - 4$ at the point where this curve cuts the line $x = 5$. Ans: $y = 7x - 29$

7. Find the equation of the tangent to the curve $y = (2x-3)(x-1)$ at each of the points where this curve cuts the x- axis. Find the point of intersection of these tangents.

   Ans: $y + x = 1$, $2y = 2x - 3; (\frac{5}{4}, -\frac{1}{4})$

8. Find the equation of the normal to the curve $y=x^2 - 6x + 5$ at each of the points where the curve cuts the x- axis. Ans: $4y - x + 1 = 0$, $4y + x - 5 = 0$

9. Find the equation of the tangent to the curve $y=x^2 + 5x - 3$ at the points where the line $y = x + 2$ crosses the curve. Ans: $y = 7x - 4$, $y + 5x + 28 = 0$

10. Find the coordinates of the point on the curve $y=2x^2$ at which the gradient is 8 Hence find the equation of the tangent to $y=2x^2$whose gradient is 8. Ans: (2 ,8),$y = 8x - 8$

11. Find the coordinates of the point on the curve $3x^2 - 1$ at which the gradient is 3. Ans: $(\frac{1}{2}, -\frac{1}{4})$

12. Find the equation of the tangent to the curve $y = 2x^2 - 2x + 1$ which has a gradient of 0.5

    Ans :$2y = x + 2$

13. Find the value of k for which $y = 2x + k$ is a tangent to the curve $y = 2x^2 - 3$. Ans $k = -\frac{7}{2}$

14. Find the equation of the tangent to the curve $y = (x-5)(2x + 1)$ which is parallel to the x-axis. Ans: $8y + 121 = 0$

15. A curve has the equation $y = x^3 - px + q$. The tangent to the curve at the point (2,-8) is parallel to the x-axis. Find the values of p and q. find also the coordinates of the other point where the tangent is parallel to the x-axis. Ans p=12,q=8;(-2,24)

16. The function $ax^2 + bx + c$ has a gradient function $4x + 2$ and a stationary value of 1. Find the values of a, b and c. ***Ans a=2 , b=2 and c=*** $\frac{3}{2}$

17. Find the second differential of y with respect to x for each of the following :

    a) $y = 6x^2 + 7$

    b) $y = 5x^3 + 6x - 5$

    c) $y = 2 + \frac{3}{x}$

18. If $y = 3x^2 - x$ show that $y\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y + 1 = 6x$.

19. The tangent to the curve $y = ax^2 + bx + 2$ at $(1 , \frac{1}{2})$ is parallel to the normal to the curve $y = x^2 + 6x + 10$ at (-2, 2). Find the values of a and b. Ans: 1 ,-2.5

20. Find the coordinates of any stationary points on the given curves and distinguish between them.

    a) $y = 2x^2 - 8x$

    b) $y = x^3 - x^2 - x + 7$

    c) $y = 1 - 3x + x^3$

    d) $y = (x-1)(x^2 - 6x + 2)$

    e) $y = 18x - 20 - 3x^3$

    f) $y = x^3 + 6x^2 + 12x + 12$

    g) $y = x^3 - 3x^2 + 3x - 1$

21. Find the coordinates of the stationary points on the following curves and distinguish between them. Hence sketch the curves.

    a) $y = x^4 + 2x^3$

    b) $y = x^3 - 4x^2 + 4x$

    c) $y = 5x^6 - 12x^5$

d) $y = x^4 - 4x^3 + 4x^2$

22. Differentiate $x^2 + \frac{1}{x}$ from first principles.

23. Differentiate $y = \frac{x}{x^2+1}$ with respect to x from first principles

24. Find the derivative of $\frac{1}{\sqrt{x}}$ from first principles

25. Find the equation of the normal to the curve $y = x^2 + 5x + 3$ that is parallel to the line

    $y = 9x$.

26. Differentiate $P = x - x^2 + \frac{\pi}{2x}$ with respect to x where π is a constant

27. Find the equation of a tangent to the curve $y=2-4^{x2}+^{x3}$ at a point (1,-1)

28. Find the stationary points of the curve $y= 5+24x-9x^2-2x^3$ and distinguish the nature of these stationary points.

29. P and Q are neighboring points on the curve $y = 2(x - x^2)$. P is the point (x,y) and Q the point $(x + \delta x, y + \delta y)$. Find the value of the ratio $\frac{\delta y}{\delta x}$ and determine the gradient of the curve at point P.

30. Differentiate the following using the first principles.

    a) $y = x^3 + x^2$

    b) $y = \frac{1}{x^2}$

    c) $y = \frac{1}{2x^2}$

31. P is the point (x,y) and Q the point $(x + \delta x, y + \delta y)$. On the graph of $y = \sqrt{x}$. Show that $\frac{\delta y}{\delta x} = \frac{1}{\sqrt{(x+\delta x)} + \sqrt{x}}$. And hence find the gradient of the curve at the point P.

32. Find the slope of the curve $y = ax^2 + bx + c$, where a,b and c are constants at the point whose x coordinate is x. At what point is the tangent to the curve parallel to the x- axis?

33. Find the gradient of the curve $y = 9x - x^3$ at the point where $x = 1$. Find the equation of the tangent to the curve at this point. Where does this tangent meet the line $y = x$?

34. Find the equation of the tangent at the point (2,4) to the curve $y = x^3 - 2x$. Also find the coordinates of the point where the tangent meets the curve again.

35. Find the equation of the tangent to the curve $y = x^3 - 9x^2 + 20x - 8$ at the point (1,4). At what points of the curve is the tangent parallel to the line $4x + y - 3 = 0$?

36. Find the equation of the tangent to the curve $y = x^3 + \frac{1}{2}x^2 + 1$ at the point $(-1, \frac{1}{2})$. Find the coordinates of another point on the curve where the tangent is parallel to that at the point $(-1, \frac{1}{2})$.

37. Find the points of intersection with the x-axis of the curve $y = x^3 - 3x^2 + 2x$,and find the equation of the tangent to the curve at each of these points.

38. Find the equations of the normals to the parabola $4y=x^2$ at the points (-2,1) and (-4,4). Show that the point of intersection of these two points lies on the parabola.

39. Find the equation of the tangent at the point (1,-1) to the curve $y = 2 - 4x^2 + x^3$. What are the coordinates of the point where the tangent meets the curve again? Find the equation of the tangent at this point.

40. Find the coordinates of the point P on the curve $8y=4-x^2$ at which the gradient is $\frac{1}{2}$. Write down the equation of the tangent to the curve at P. find also the equation of the tangent to the curve whose gradient is $-\frac{1}{2}$, and the coordinates of its point of intersection with the tangent at P.

41. Find the equations of the tangents to the curve $y = x^3 - 6x^2 + 12x + 2$ which are parallel to the line $y = 3x$.

42. Find the coordinates of the points of intersection of the line $x - 3y = 0$ with the curve $y = x(1-x^2)$. If these points are in order P,O,Q, prove that the tangents to the curve at P and Q are parallel, and that the tangent at O is perpendicular to them.

43. Find the equations of the tangent and the normal to the parabola $x^2 = 4y$ at the point (6,9). Also find the distance between the points where the tangent and the normal meet the y-axis.

44. The curve $y = (x-2)(x-4)(x-3)$ cuts the x-axis at the points P(2,0) , Q(3,0), R(4,0). Prove that the tangents at P and R are parallel. At what point does the normal to the curve at Q cut the y-axis?

45. Find the equation of the tangent at the point P(3,9) to the curve $y = x^3 - 6x^2 + 15x - 9$

    If O is the origin and N is the foot of the perpendicular from P to the x-axis, prove that the tangent at P passes through the mid-point of ON. Find the coordinates of another point on the curve, the tangent at which is parallel to the tangent at the point (3,9).

46. A tangent to the parabola $x^2 = 16y$ is perpendicular to the line $x - 2y - 3 = 0$. Find the equation of this tangent and the coordinates of its point of contact.

47. Find the equation of the tangent to $y = x^2$ at the point (1,1) and of the tangent to $y = \frac{1}{6}x^3$ at the point $(2,\frac{4}{3})$. Show that these tangents are parallel, and find the distance between them.

48. The curve C is defined by $y = ax^2 + b$, where a and b are constants. Given that the gradient of the curve at the point (2,-2) is 3, find the values of a and b.

49. Given that the curve with equation $y = Ax^2 + Bx$ has gradient 7 at the point (6,8), find the values of the constants A and B.

50. A curve with equation $y = A\sqrt{x} + \frac{B}{\sqrt{x}}$, for constants A and B, passes through the point (1,6) with gradient -1. Find A and B.

51. Find the equation of the tangent, t, to the curve $y = x^2 + 5x + 2$, which is perpendicular to the line, l, with equation $3y + x = 5$.

END